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TORSIONAL IMPACT RESPONSE OF AN AXISYMMETRIC INTERNAL OR EDGE CRACK IN AN ELASTIC SOLID WITH A CYLINDRICAL CAVITY

YASUHIDE SHINDO, WEI LI, YOSHIHISA KANNO and AKIRA ATSUMI
Department of Mechanical Engineering II, Faculty of Engineering, Tohoku University, Sendai 980, Japan

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Abstract—The torsional impact response of a flat annular or circumferential edge crack in an infinite elastic medium with a cylindrical cavity is investigated. Laplace and Hankel transforms are used to reduce the problem to a set of integral equations. These equations are solved numerically and the singular stress field near the crack tip is determined. The influence of geometry and inertia upon the stress intensity factor is shown graphically in detail.

1. INTRODUCTION

The impact response of a crack has been a subject of recent interest. For an axisymmetric geometry, the transient response of a flat annular crack in a finite elastic cylinder under a torsional load has been considered (Shindo, 1982). In studying the fracture problem in pressure vessels, pipes and other cylindrical containers, if the crack around the inner boundary is sufficiently small, then the effect of the outer boundary can be neglected as a first approximation (Anzai *et al.*, 1981).

Thus the present analysis considers the linear elastodynamic fracture analysis of a cylindrical cavity having a flat annular or circumferential edge crack, which is subjected to a torsional impact load. Laplace and Hankel transforms are used to reduce the mixed boundary value problem to a set of integral equations. The solution is then expressed in terms of a singular integral equation of the first kind having the kernel with a fast rate of convergence. The solution of the singular integral equation is expanded in terms of the Chebyshev polynomials for the internal crack or the Jacobi polynomials for the edge crack and obtained by solving an infinite set of linear algebraic equations (Erdogan *et al.*, 1973). A numerical Laplace inversion technique (Papoulis, 1957) is used to recover the time dependence of the solution. As the static results have not been reported yet, we discuss the static problem here, too. The dynamic stress intensity factor is computed and the numerical values are shown in graphs for various geometrical parameters at designated time instances.

2. FORMULATION OF THE PROBLEM

Let an infinite homogeneous isotropic medium with a cylindrical cavity of radius a, as shown in Fig. 1, be subjected to torsional load. The axis of the cylindrical cavity is assumed

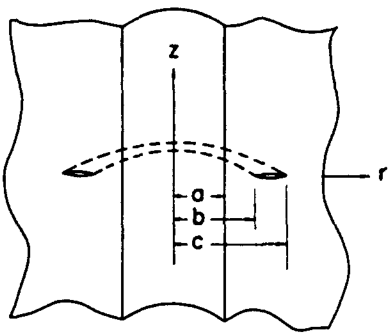

Fig. 1. Geometry and coordinate system.

to coincide with the z-axis of the cylindrical coordinate system (r, θ, z). Let μ and ρ be the shear modulus of the medium and the mass density, respectively. A flat annular crack of inner radius b and outer radius c lies in the plane $z = 0$ with its center at the origin of the coordinate axes. Let the components of the displacements in the r-, θ- and z-directions be denoted by u_r, u_θ and u_z, respectively. For torsional motion, u_r and u_z vanish everywhere and u_θ is a function of r, z and time t only. The displacement field can thus be written as

$$u_r = u_z = 0, \quad u_\theta = u_\theta(r, z, t). \tag{1}$$

The stress field corresponding to eqn (1) is

$$\tau_{r\theta} = \mu\left(\frac{\partial u_\theta}{\partial r} - \frac{1}{r}u_\theta\right)$$

$$\tau_{\theta z} = \mu\frac{\partial u_\theta}{\partial z}. \tag{2}$$

Under these considerations, the equation of motion can be written as

$$\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} + \frac{\partial^2 u_\theta}{\partial^2 z} = \frac{1}{c_2^2}\frac{\partial^2 u_\theta}{\partial t^2} \tag{3}$$

where $c_2 = \sqrt{(\mu/\rho)}$ is the shear wave velocity.

For the present case, the initial conditions are all taken to be zero. The sudden torque applied to the medium generates torsional waves normally incident on the crack. By the principle of superposition, the equivalent boundary conditions for which the wave passes across the crack plane at $z = 0$ can be written as

$$\tau_{\theta z}(r, 0, t) = -\tau_0\frac{r}{c}H(t), \quad b < r < c$$

$$u_\theta(r, 0, t) = 0, \quad a \leqslant r \leqslant b, \quad c \leqslant r \tag{4}$$

$$\tau_{r\theta}(a, z, t) = 0 \tag{5}$$

where $H(t)$ is a Heaviside unit step function and τ_0 is a constant with the dimensions of stress. In addition to eqns (4) and (5), all components of displacement and stress vanish at remote distances from the crack region.

A Laplace transform pair is defined by

$$f^*(p) = \int_0^\infty f(t)\,e^{-pt}\,dt$$

$$f(t) = \frac{1}{2\pi i}\int_{Br} f^*(p)\,e^{pt}\,dp \tag{6}$$

where Br stands for the Bromwich path of integration. Applying the Laplace transform to eqn (3), we obtain

$$\frac{\partial^2 u_\theta^*}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta^*}{\partial r} - \frac{u_\theta^*}{r^2} + \frac{\partial^2 u_\theta^*}{\partial z^2} = \left(\frac{p}{c_2}\right)^2 u_\theta^*. \tag{7}$$

The boundary conditions in the Laplace transform plane are

$$\tau^*_{\theta z}(r,0,p) = -\frac{\tau_0 r}{cp}, \quad b < r < c$$

$$u^*_\theta(r,0,p) = 0, \quad a \leqslant r \leqslant b, \quad c \leqslant r \tag{8}$$

$$\tau^*_{r\theta}(a,z,p) = 0. \tag{9}$$

Now using the integral transform, we obtain a solution of eqn (7)

$$u^*_\theta(r,z,p) = \int_0^\infty A(s,p)J_1(sr)\,\mathrm{e}^{-\gamma z}\,\mathrm{d}s + \int_0^\infty C(s,p)K_1(\gamma r)\sin(sz)\,\mathrm{d}s \tag{10}$$

where $J_n(\ )$ and $K_n(\ )$ are usual n-order Bessel and modified Bessel functions, respectively. $A(s,p)$ and $C(s,p)$ are unknown functions to be determined and

$$\gamma(s,p) = \left\{s^2 + \left(\frac{p}{c_2}\right)^2\right\}^{1/2}.$$

The Laplace transforms of the stress components are

$$\tau^*_{r\theta}(r,z,p) = -\mu\int_0^\infty sA(s,p)J_2(sr)\,\mathrm{e}^{-\gamma z}\,\mathrm{d}s - \mu\int_0^\infty \gamma C(s,p)K_2(\gamma r)\sin(sz)\,\mathrm{d}s$$

$$\tau^*_{\theta z}(r,z,p) = -\mu\int_0^\infty \gamma A(s,p)J_1(sr)\,\mathrm{e}^{-\gamma z}\,\mathrm{d}s + \mu\int_0^\infty sC(s,p)K_1(\gamma r)\cos(sz)\,\mathrm{d}s. \tag{11}$$

Substituting eqn $(11)_1$ into eqn (9), we obtain

$$-\int_0^\infty sA(s,p)J_2(sa)\,\mathrm{e}^{-\gamma z}\,\mathrm{d}s - \int_0^\infty \gamma C(s,p)K_2(\gamma a)\sin(sz)\,\mathrm{d}s = 0. \tag{12}$$

Applying the Fourier sine transform to eqn (12), we obtain

$$\gamma K_2(\gamma a)C(s,p) = -\frac{2s}{\pi}\int_0^\infty \frac{\eta}{\eta^2+\gamma^2}A(\eta)J_2(\eta a)\,\mathrm{d}\eta. \tag{13}$$

Making use of eqns (10) and $(11)_2$ and conditions (8) leads to the following triple integral equations:

$$-\int_0^\infty \gamma A(s,p)J_1(sr)\,\mathrm{d}s + \int_0^\infty sC(s,p)K_1(\gamma r)\,\mathrm{d}s = -\frac{\tau_0 r}{\mu cp} \quad (b < r < c)$$

$$\int_0^\infty A(s,p)J_1(sr)\,\mathrm{d}s = 0 \quad (a \leqslant r \leqslant b,\ c \leqslant r). \tag{14}$$

In order to solve the triple integral equations, eqns (14), we introduce the representation

$$A(s,p) = -\int_b^c q\phi^*(q,p)J_2(sq)\,\mathrm{d}q. \tag{15}$$

For the sake of convenience, we perform the following nondimensionalization and change of variables:

$$a_0 = a/c, \quad b_0 = b/c$$
$$r_0 = r/c = \tfrac{1}{2}\{(1-b_0)\zeta+1+b_0\}$$
$$q_0 = q/c = \tfrac{1}{2}\{(1-b_0)\tau+1+b_0\}$$
$$P = pc/c_2$$
$$\Phi^*(\tau, P) = \phi^*(q, p)\frac{\mu p}{\tau_0 q_0}. \tag{16}$$

Substituting eqns (13) into eqns (14) and making use of eqns (15) and (16), we find that

$$\int_{-1}^{1} \frac{1-b_0}{2}\frac{q_0^2}{r_0}\{R_0(r_0, q_0)+R_1(r_0, q_0)+R_2(r_0, q_0)\}\Phi^*(\tau, P)\,\mathrm{d}\tau = -1 \quad (b < r < c)$$
$$\int_{-1}^{1} \Phi^*(\tau, P)\,\mathrm{d}\tau = 0 \quad (a \leqslant r \leqslant b) \tag{17}$$

where

$$R_0(r_0, q_0) = \int_0^{\infty} sJ_2(sq_0)J_1(r_0 s)\,\mathrm{d}s \tag{18}$$

$$R_1(r_0, q_0) = \int_0^{\infty} (\gamma_0 - s)J_2(sq_0)J_1(r_0 s)\,\mathrm{d}s \tag{19}$$

$$R_2(r_0, q_0) = \frac{2}{\pi}\int_0^{\infty} \frac{s^2}{\gamma_0}\frac{K_1(\gamma_0 r_0)I_2(\gamma_0 a_0)K_2(\gamma_0 q_0)}{K_2(\gamma_0 a_0)}\,\mathrm{d}s \tag{20}$$

$$\gamma_0 = (s^2+P^2)^{1/2}. \tag{21}$$

The kernel function $R_0(r_0, q_0)$ has Cauchy type and logarithmic singularities. Separating the singularities from the function, we obtain

$$\pi q_0^2 R_0(r_0, q_0) = \frac{2r_0}{1-b_0}\left\{\frac{1}{\tau-\zeta} - \frac{3(1-b_0)}{4r_0}\log|\tau-\zeta| + \frac{1-b_0}{2r_0}M_0(r_0, q_0)\right\} \tag{22}$$

where $M_0(r_0, q_0)$ is the Fredholm kernel without the singularities which is given by

$$\begin{aligned} M_0(r_0, q_0) &= \left\{\frac{r_0^2}{(q_0+r_0)q_0} - 2\right\}E\left(\frac{q_0}{r_0}\right) + \frac{r_0}{q_0-r_0}\left\{\frac{r_0}{q_0}E\left(\frac{q_0}{r_0}\right) - 1\right\} \\ &\quad + 4K\left(\frac{q_0}{r_0}\right) + \tfrac{3}{2}\log|\tau-\zeta| \quad (q_0 < r_0) \\ &= \left\{\frac{r_0^2}{q_0-r_0} - 2\frac{q_0}{r_0}\right\}E\left(\frac{r_0}{q_0}\right) + \frac{r_0}{q_0-r_0}\left\{E\left(\frac{r_0}{q_0}\right) - 1\right\} \\ &\quad + 2\frac{q_0}{r_0}K\left(\frac{r_0}{q_0}\right) + \tfrac{3}{2}\log|\tau-\zeta| \quad (q_0 > r_0). \end{aligned} \tag{23}$$

$K(\ )$ and $E(\ )$ are the complete elliptic integrals of the first and second kind, respectively.

Substituting eqn (22) into $(17)_1$, we obtain the singular integral equation of the first kind

$$\frac{1}{\pi}\int_{-1}^{1}\left\{\frac{1}{\tau-\zeta}-\frac{3(1-b_0)}{4r_0}\log|\tau-\zeta|+L(r_0,q_0)\right\}\Phi^*(\tau,P)\,d\tau=-1 \tag{24}$$

where

$$L(r_0,q_0)=\frac{1-b_0}{2r_0}[M_0(r_0,q_0)+\pi q_0^2\{R_1(r_0,q_0)+R_2(r_0,q_0)\}]. \tag{25}$$

The kernel function $R_1(r_0,q_0)$ is a semi-infinite integral with a slow rate of convergence. Using the contour integration technique (Shindo, 1981), eqn (19) can be written as

$$\begin{aligned}\pi q_0^2R_1(r_0,q_0)&=2P^2q_0^2\left[\int_0^1 sI_2(sPq_0)K_1(sPr_0)\,ds\right.\\&\left.+\int_1^\infty\{s-(s^2-1)^{1/2}\}I_2(sPq_0)K_1(sPr_0)\,ds\right]\quad(q_0<r_0)\\&=-2P^2q_0^2\left[\int_0^1 sK_2(sPq_0)I_1(sPr_0)\,ds\right.\\&\left.+\int_1^\infty\{s-(s^2-1)^{1/2}\}K_2(sPq_0)I_1(sPr_0)\,ds\right]+\pi Pr_0\quad(q_0>r_0).\end{aligned} \tag{26}$$

The kernel function $R_2(r_0,q_0)$ is also a semi-infinite integral with a slow rate of convergence for $r_0,q_0\to b_0\to a_0$. Thus, $R_2(r_0,q_0)$ is modified as follows:

$$\begin{aligned}\pi q_0^2R_2(r_0,q_0)=2q_0^2\int_0^\infty\left\{\frac{s^2}{\gamma_0}\frac{K_1(\gamma_0r_0)I_2(\gamma_0a_0)K_2(\gamma_0q_0)}{K_2(\gamma_0a_0)}-A_1^*(s,r_0,q_0)\right\}ds\\+2q_0^2\int_0^\infty A_1^*(s,r_0,q_0)\,ds\end{aligned} \tag{27}$$

where

$$A_1^*(s,r_0,q_0)=a_1\,e^{-(r_0+q_0-2a_0)s}+\frac{b_1}{s}\{e^{-(r_0+q_0-2a_0)s}-e^{-\delta_1s}\}$$

$$\int_0^\infty A_1^*(s,r_0,q_0)\,ds=\frac{a_1}{r_0+q_0-2a_0}+b_1\log\left(\frac{\delta_1}{r_0+q_0-2a_0}\right) \tag{28}$$

$$a_1=\frac{1}{2(r_0q_0)^{1/2}}$$

$$b_1=\frac{a_1}{8}\left\{\frac{3}{r_0}+\frac{15}{q_0}-\frac{30}{a_0}-4(r_0+q_0-2a_0)P^2\right\} \tag{29}$$

$$\delta_1=q_0+r_0. \tag{30}$$

3. SOLUTION OF THE INTERNAL CRACK PROBLEM

The internal crack problem is now reduced to the solution of eqn (24) under $(17)_2$. The solution of the singular integral equation of the first kind can be put in the form (Shindo, 1982; Erdogan *et al.*, 1973)

$$\Phi^*(\tau, P) = \frac{1}{(1-\tau^2)^{1/2}} \sum_{n=1}^{\infty} A_n^*(P) T_n(\tau) \tag{31}$$

where $T_n(\ )$ are the n-order Chebyshev polynomials of the first kind, $A_n^*(P)$ are unknown constants and eqn $(17)_2$ is identically satisfied by eqn (31). Substituting eqn (31) into eqn (24) and making use of the orthogonality of the Chebyshev polynomials, we obtain an infinite system of linear algebraic equations

$$\sum_{n=1}^{\infty} A_n^*(P)[\delta_{kn} + \alpha_{kn} + \beta_{kn}] = -\delta_{1k} \quad (k = 1, 2, \ldots) \tag{32}$$

where

$$\alpha_{kn} = \frac{3(1-b_0)}{2n\pi} \int_{-1}^{1} \frac{1}{r_0} T_n(\zeta) U_{k-1}(\zeta)(1-\zeta^2)^{1/2}\, d\zeta$$

$$\beta_{kn} = \frac{2}{\pi^2} \int_{-1}^{1} U_{k-1}(\zeta)(1-\zeta^2)^{1/2}\, d\zeta \int_{-1}^{1} T_n(\tau) \frac{1}{(1-\tau^2)^{1/2}} L(\zeta, \tau)\, d\tau \tag{33}$$

δ_{kn} is the Kronecker delta and $U_{k-1}(\ )$ are the $(k-1)$-order Chebyshev polynomials of the second kind. All integrals in eqns (33) are of the Gauss–Chebyshev type and may be evaluated easily by using the proper quadrature formulas (Erdogan *et al.*, 1973).

The singular stress field can be found as

$$\tau_{r\theta} \sim \frac{K_{3b}(T)}{(2\rho_b)^{1/2}} \cos\left(\frac{\theta_b}{2}\right) - \frac{K_{3c}(T)}{(2\rho_c)^{1/2}} \sin\left(\frac{\theta_c}{2}\right)$$

$$\tau_{\theta z} \sim \frac{K_{3b}(T)}{(2\rho_b)^{1/2}} \sin\left(\frac{\theta_b}{2}\right) + \frac{K_{3c}(T)}{(2\rho_c)^{1/2}} \cos\left(\frac{\theta_c}{2}\right) \tag{34}$$

where $T = c_2 t/c$ is the normalized time and (ρ_b, θ_b) and (ρ_c, θ_c) are the polar coordinates centered at the crack border in the plane $z = 0$

$$\rho_b = \{(r-b)^2 + z^2\}^{1/2}, \quad \theta_b = \tan^{-1}\left\{\frac{z}{r-b}\right\}$$

$$\rho_c = \{(r-c)^2 + z^2\}^{1/2}, \quad \theta_c = \tan^{-1}\left\{\frac{z}{r-c}\right\}. \tag{35}$$

The dynamic stress intensity factors $K_{3b}(T)$ and $K_{3c}(T)$ are given by

$$\frac{K_{3b}(T)}{\tau_0 c^{1/2}} = b_0\left(\frac{1-b_0}{2}\right)^{1/2} \frac{1}{2\pi i} \int_{Br} \sum_{n=1}^{\infty} \frac{1}{P} (-1)^n A_n^*(P)\, e^{PT}\, dP$$

$$\frac{K_{3c}(T)}{\tau_0 c^{1/2}} = -\left(\frac{1-b_0}{2}\right)^{1/2} \frac{1}{2\pi i} \int_{Br} \sum_{n=1}^{\infty} \frac{1}{P} A_n^*(P)\, e^{PT}\, dP. \tag{36}$$

4. SOLUTION OF THE EDGE CRACK PROBLEM

In this section, we consider the circumferential edge crack problem for $a = b$ which is reduced to the solution of eqn (24). The solution of the singular integral equation of the first kind can be expanded in the form (Anzai *et al.*, 1981; Erdogan *et al.*, 1973)

$$\Phi^*(\tau, P) = \sum_{n=0}^{\infty} B_n^*(P) P_n^{(-1/2,1/2)}(\tau) \left(\frac{1+\tau}{1-\tau}\right)^{1/2} \tag{37}$$

where $P_n^{(-1/2,1/2)}(\ )$ denote the n-order Jacobi polynomials and $B_n^*(P)$ are unknown constants. Substituting eqn (37) into eqn (24) and making use of the orthogonality of the Jacobi polynomials, we have

$$\theta_k^{(1/2,-1/2)} B_k^*(P) + \sum_{n=0}^{\infty} (G_{kn} + C_{kn}) B_n^*(P) = -\pi \delta_{k0} \frac{a}{c} \quad (k = 0, 1, 2, \ldots) \tag{38}$$

where

$$\theta_0^{(1/2,-1/2)} = \pi$$

$$\theta_k^{(1/2,-1/2)} = \frac{2(2k)!\,(2k+2)!\,\pi}{(1+2k)(k!)^3(k+1)!\,2^{2(k+1)}} \quad (k = 1, 2, \ldots) \tag{39}$$

$$G_{kn} = \int_{-1}^{1} P_k^{(1/2,-1/2)}(\zeta) W_n(\zeta) \left(\frac{1-\zeta}{1+\zeta}\right)^{1/2} \mathrm{d}\zeta \tag{40}$$

$$(k, n = 0, 1, 2, \ldots)$$

$$C_{kn} = \int_{-1}^{1} P_k^{(1/2,-1/2)}(\zeta) V_n(\zeta) \left(\frac{1-\zeta}{1+\zeta}\right)^{1/2} \mathrm{d}\zeta \tag{41}$$

$$W_0(\zeta) = \frac{3}{4r_0}(\zeta + \log 2)$$

$$W_n(\zeta) = \frac{3(2n)!}{4r_0(n!)^2 2^{2n}} \left\{\frac{T_n(\zeta)}{n} + \frac{T_{n+1}(\zeta)}{n+1}\right\} \quad (n = 1, 2, \ldots) \tag{42}$$

$$V_n(\zeta) = \frac{1}{\pi} \int_{-1}^{1} L(\zeta, \tau) P_n^{(-1/2,1/2)}(\tau) \left(\frac{1+\tau}{1-\tau}\right)^{1/2} \mathrm{d}\tau \quad (n = 0, 1, 2, \ldots). \tag{43}$$

In eqns (40), (41) and (43), all integrals are of the Gauss–Jacobi type and may be also evaluated by using the proper quadrature formulas (Erdogan *et al.*, 1973).

Following the similar method in the internal crack problem, the singular stress field in this case is also obtained as

$$\tau_{r\theta} \sim -\frac{K_3(T)}{(2\rho_c)^{1/2}} \sin\left(\frac{\theta_c}{2}\right)$$

$$\tau_{\theta z} \sim \frac{K_3(T)}{(2\rho_c)^{1/2}} \cos\left(\frac{\theta_c}{2}\right). \tag{44}$$

The dynamic stress intensity factor $K_3(T)$ is given by

$$\frac{K_3(T)}{\tau_0 c^{1/2}} = -\frac{\{2(1-a_0)\}^{1/2}}{a_0} \frac{1}{2\pi \mathrm{i}} \int_{Br} \sum_{n=0}^{\infty} \frac{1}{P} B_n^*(P) P_n^{(-1/2,1/2)}(1)\, \mathrm{e}^{PT}\, \mathrm{d}P. \tag{45}$$

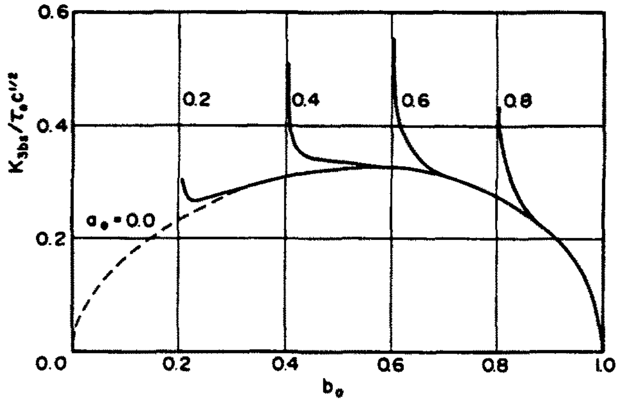

Fig. 2. Normalized static stress intensity factor $K_{3b}/\tau_0 c^{1/2}$ vs b_0 for various a_0 ratios.

5. NUMERICAL RESULTS AND CONSIDERATION

An infinite system of algebraic equations, eqns (32) and (38), is solved numerically and the values of $A_n^*(P)$ and $B_n^*(P)$ are obtained for discrete values of P. Then, using the Laplace inversion technique developed by Papoulis (1957), the numerical values of $K_{3b}(T)/\tau_0 c^{1/2}$, $K_{3c}(T)/\tau_0 c^{1/2}$ and $K_3(T)/\tau_0 c^{1/2}$ are obtained from eqns (36) and (45).

As $T \to \infty$, the dynamic stress intensity factors tend to the static solutions. If we use the Final-value theorem, we can obtain the static stress intensity factors as follows:

$$\frac{K_{3bs}}{\tau_0 c^{1/2}} = \lim_{P \to 0} b_0 \left(\frac{1-b_0}{2}\right)^{1/2} \sum_{n=1}^{\infty} A_n^*(P)(-1)^n \tag{46}$$

$$\frac{K_{3cs}}{\tau_0 c^{1/2}} = -\lim_{P \to 0} \left(\frac{1-b_0}{2}\right)^{1/2} \sum_{n=1}^{\infty} A_n^*(P) \tag{47}$$

$$\frac{K_{3s}}{\tau_0 c^{1/2}} = -\lim_{P \to 0} \frac{\{2(1-a_0)\}^{1/2}}{a_0} \sum_{n=0}^{\infty} B_n^*(P) P_n^{(1/2,-1/2)}(1). \tag{48}$$

Figure 2 shows the variation of normalized static stress intensity factors $K_{3bs}/\tau_0 c^{1/2}$ at the inner tip of the crack against the ratio b_0 for various values of the ratio a_0. The same kind of results for K_{3cs} at the outer tip of the crack is shown in Fig. 3. The dashed curves obtained

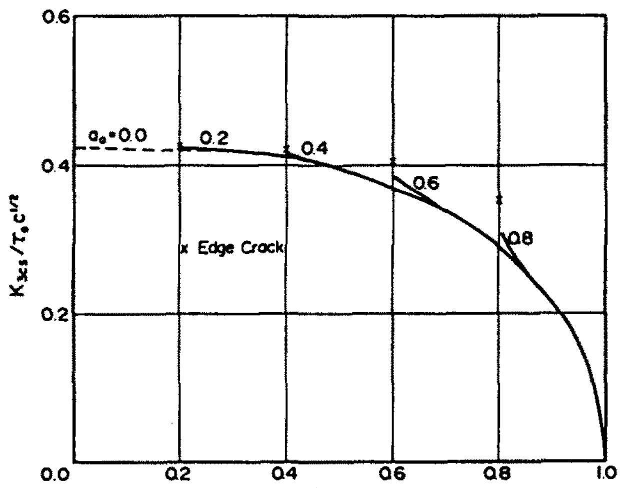

Fig. 3. Normalized static stress intensity factor $K_{3c}/\tau_0 c^{1/2}$ vs b_0 for various a_0 ratios.

Table 1. Normalized static stress intensity factors $K_{3bs}/\tau_0 c^{1/2}$ and $K_{3cs}/\tau_0 c^{1/2}$ for b_0 approaching a_0

a_0	b_0	$K_{3bs}/\tau_0 c^{1/2}$	$K_{3cs}/\tau_0 c^{1/2}$
	0	—	0.4244
0	0.1	0.1697	0.4239
	0.15	0.2062	0.4236
	0.2	—	0.4241
0.2	0.21	0.3028	0.4224
	0.22	0.2779	0.4219
	0.4	—	0.4210
0.4	0.405	0.5085	0.4135
	0.41	0.4338	0.4128
	0.6	—	0.4055
0.6	0.605	0.5533	0.3854
	0.61	0.4644	0.3818
	0.8	—	0.3494
0.8	0.805	0.4343	0.3091
	0.81	0.3635	0.3013

for the case of $a_0 = 0$ coincide with the static case of a flat annular crack in an infinite medium. The static stress intensity factor K_{3bs} for $a_0 = 0$ tends to zero and K_{3cs} for $a_0 = 0$ tends to the solution $(4/3\pi)\tau_0 c^{1/2}$ for a penny-shaped crack as $b_0 \to 0$. It can be seen that both K_{3bs} and K_{3cs} for $a_0 = 0.4, 0.6, 0.8$ increase as the ratio b_0 decreases. As the inner crack tip approaches the surface of the cylindrical cavity, the values of $K_{3bs}/\tau_0 c^{1/2}$ increase rapidly. In Table 1, the values of $K_{3bs}/\tau_0 c^{1/2}$ and $K_{3cs}/\tau_0 c^{1/2}$ are listed for the b_0 ratio approaching a_0. The computation for $a_0 < b_0$ is based on the internal crack solution in Section 3 and we cannot obtain the case of the edge crack by simply taking b_0 to be a_0. The numerical result for $b_0 = a_0$ is based on the edge crack solution in Section 4. As $b_0 \to a_0$, the values of $K_{3bs}/\tau_0 c^{1/2}$ and $K_{3cs}/\tau_0 c^{1/2}$ do not tend to the results for a circumferential edge crack. This is also shown for a half-plane with an internal or edge crack by Kaya and Erdogan (1987).

Figures 4 and 5 show the influence of the ratio a_0 on the transient stress intensity factors $K_{3b}/\tau_0 c^{1/2}$ and $K_{3c}/\tau_0 c^{1/2}$ with the time variable T for the ratio $b_0 = 0.5$, respectively. The dashed curves for $a_0 = 0$ are the results for the case of a flat annular crack in an infinite medium (Shindo, 1981). As a_0 is increased, the peak values of $K_{3c}/\tau_0 c^{1/2}$ occur at a later time. The dynamic stress intensity factor $K_{3c}/\tau_0 c^{1/2}$ tends to the solution of a circumferential edge crack in a cylindrical cavity as $a_0 \to 0.5$. The peak values of $K_{3b}/\tau_0 c^{1/2}$ and $K_{3c}/\tau_0 c^{1/2}$ appear to be higher for larger a_0 ratios. It is clear that the effect of a cylindrical cavity

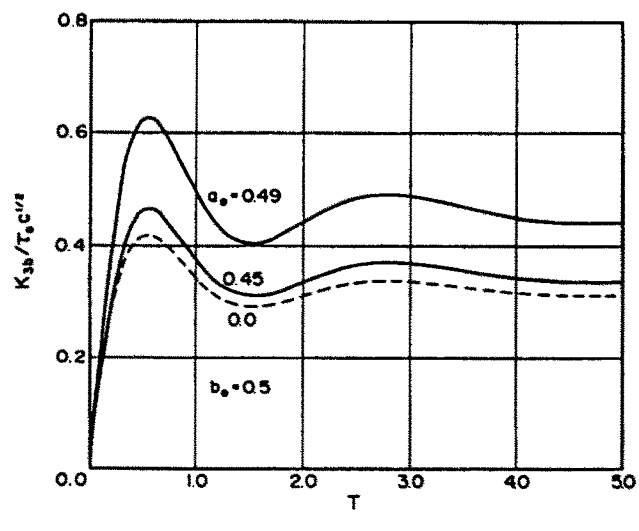

Fig. 4. Normalized dynamic stress intensity factor $K_{3b}/\tau_0 c^{1/2}$ vs T for $b_0 = 0.5$.

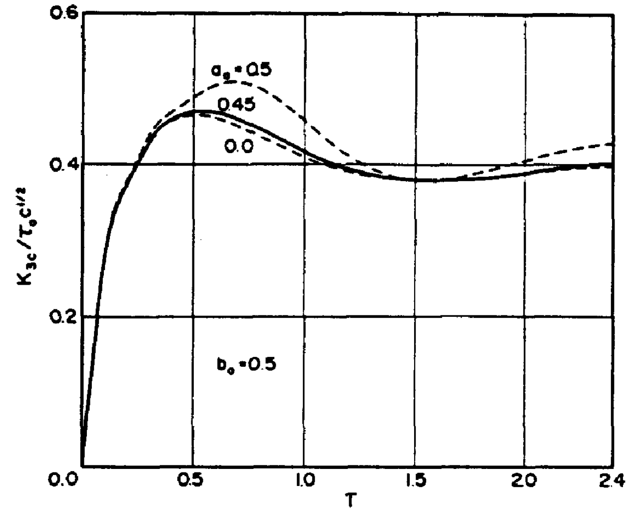

Fig. 5. Normalized dynamic stress intensity factor $K_{3c}/\tau_0 c^{1/2}$ vs T for $b_0 = 0.5$.

diminishes as $a_0 \to 0$. The effect of a cylindrical cavity on $K_{3b}/\tau_0 c^{1/2}$ is more pronounced than the effect on $K_{3c}/\tau_0 c^{1/2}$.

Figures 6 and 7 exhibit the transient stress intensity factor variations $K_{3b}/\tau_0 c^{1/2}$ and $K_{3c}/\tau_0 c^{1/2}$ with T for the ratio $a_0 = 0.5$. The peak values of $K_{3b}/\tau_0 c^{1/2}$ and $K_{3c}/\tau_0 c^{1/2}$ occur later in time and increase with the decrease of b_0.

Figure 8 exhibits the variation of the normalized dynamic stress intensity factor $K_3/\tau_0(c-a)^{1/2}$ vs $T_c = T/(1-a_0)$ for the case of a circumferential edge crack in a cylindrical cavity. As $a_0 \to 0$, $K_3/\tau_0(c-a)^{1/2}$ tends to the solution of a penny-shaped crack. As $a_0 \to 1.0$, $K_3/\tau_0(c-a)^{1/2}$ tends to the solution of a semi-infinite medium with an edge crack of length $c-a$ normal to the edge under antiplane shear load by Thau and Lu (1970). The peak values of $K_3/\tau_0(c-a)^{1/2}$ occur later in time and increase with the increase of a_0.

In conclusion, the dynamic response of an annular or circumferential edge crack in an infinite elastic solid with a cylindrical cavity is determined in this study. The results are expressed in terms of the static and dynamic stress intensity factors. The general feature is that the dynamic stress intensity factor rises rapidly with time reaching a peak and then decreases in magnitude and oscillates about its corresponding static value. The peak value of the dynamic stress intensity factor is found to depend on the geometrical parameters a and b.

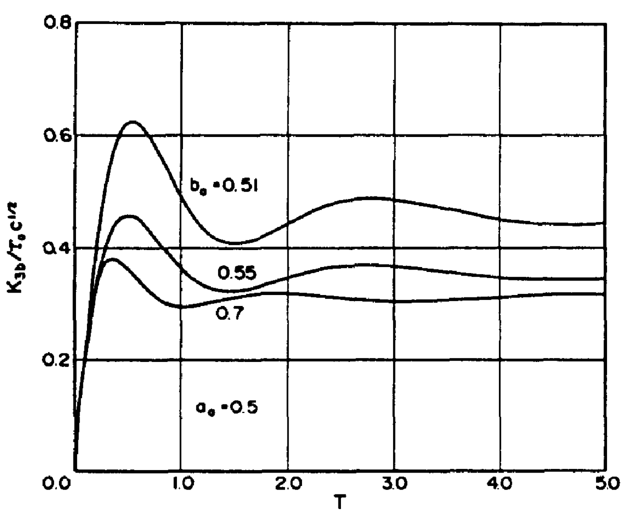

Fig. 6. Normalized dynamic stress intensity factor $K_{3b}/\tau_0 c^{1/2}$ vs T for $a_0 = 0.5$.

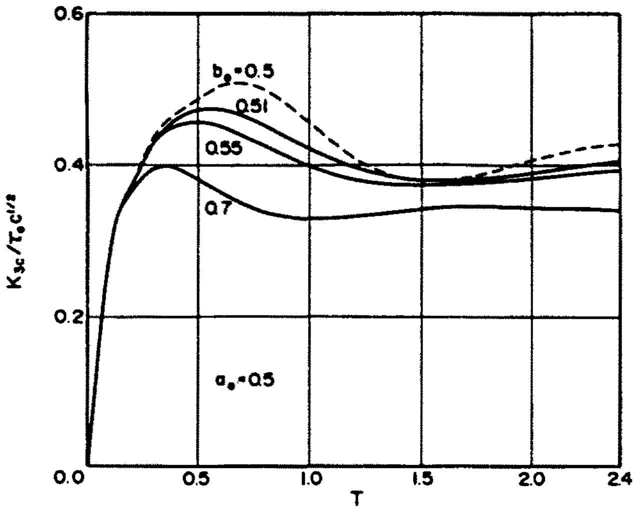

Fig. 7. Normalized dynamic stress intensity factor $K_{3c}/\tau_0 c^{1/2}$ vs T for $a_0 = 0.5$.

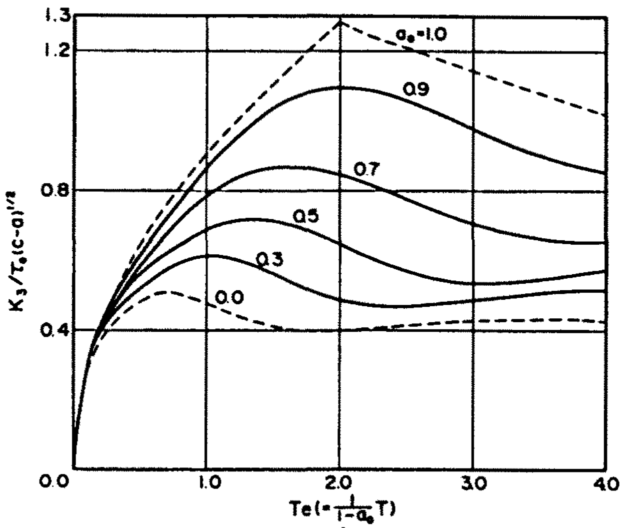

Fig. 8. Normalized dynamic stress intensity factor $K_3/\tau_0(c-a)^{1/2}$ vs T_e ($= T/(1-a_0)$) for edge crack.

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